On the Use of Synthetic Difference-in-differences Approach with (-out) Covariates: The Case Study of Brexit Referendum

Supplementary Online Appendix

Contents

Appendix A. Additional Discussion

Appendix A.1. Matching on Covariates with SDID

By analogy to the DSC method, one can also extend with matching on covariates the SDID time weights $\boldsymbol{\lambda}_t^{sdid}$.

Defining $\ddot{\boldsymbol{z}}_{T_0}$ to be the vector of the demeaned pre-treatment outcomes across the control units for the post-treatment period T_0 and the average of the covariates across control units for the same period; and denoting $\ddot{\boldsymbol{Z}}$ the matrix of stacked analogous vectors for the pre-treatment periods, the nested optimization for the time-weights is:

$$\widehat{\boldsymbol{\lambda}}^{sdid}(\boldsymbol{Q}) = \underset{\boldsymbol{\lambda}\in L}{arg\,min} \left(\ddot{\boldsymbol{z}}_{T_0} - \ddot{\boldsymbol{Z}}\boldsymbol{\lambda}\right)' \boldsymbol{Q} \left(\ddot{\boldsymbol{z}}_{T_0} - \ddot{\boldsymbol{Z}}\boldsymbol{\lambda}\right), \tag{A.1}$$

$$\widehat{\boldsymbol{Q}}^{sdid} = \underset{\boldsymbol{Q}\in Q}{arg\,min} \quad (\boldsymbol{y}_{T_0} - \boldsymbol{Y}'\widehat{\boldsymbol{\lambda}}^{sdid}(\boldsymbol{Q}))'\boldsymbol{W}_J(\boldsymbol{y}_{T_0} - \boldsymbol{Y}'\widehat{\boldsymbol{\lambda}}^{sdid}(\boldsymbol{Q})), \tag{A.2}$$

where $\mathbb{Q}$ is the set of M-dimensional diagonal positive semi-definite matrices $\boldsymbol{Q}$ where $M = J+P$ and $\boldsymbol{Q} = \text{diag}(\boldsymbol{q})$.

The counterfactual and the treatment effect estimates are obtained similarly to the case without matching on covariates.

Requirements and properties. An important feature of the SDID, as mentioned in Arkhangelsky et al. (2021), and much like the DSC, is that it borrows attractive features from both the SC and the DID methods. Intuitively, while the DSC aims at correcting for imbalances in the units, the SDID (through the time weights) also corrects for imbalances between the pre-treatment periods and the post-treatment periods. Arkhangelsky et al. (2021) claim that the introduction of time weights can both remove bias and improve precision by not taking into account time periods that are intrinsically different from post-treatment periods. Not only that but it is shown that the SDID estimator has desirable double-robustness properties. Evidently from the setting, for accurate estimation, the SDID estimator requires a setting where the number of control units J and the number of pre-treatment periods $T_0 - 1$ is large.

Appendix A.2. Gains of Matching on Covariates?

Abadie et al. (2010), Abadie et al. (2015), and Abadie (2021) recommend to estimate weights that match additionally on covariates, and only to use the SC estimator when there is a good pre-treatment fit also in terms of covariates. In particular, Abadie et al. (2010) show that matching additionally on covariates has desirable bias-reduction properties. Under the linear factor model specification, the existence of weights that perfectly match pre-treatment outcomes and covariates suggests that it is possible to obtain bounds on the bias of the SC constructed with such weights. Further, such bounds converge to zero as $T_0 \to \infty$.

This recommendation is further strengthened by Botosaru and Ferman (2017), who show that the existence of weights that provide a good fit in terms of both covariates and pre-treatment outcomes implies tighter bounds for any T_0 on the bias of the SC estimator.

However, there are natural questions that arise when one is considering to additionally match on covariates. For instance, in practice, one does not know if the covariates at hand are relevant/informative about the treatment variable $y^0_{j,t}$. Moreover, there are no clear rules of thumb or statistical procedures suggested to decide on which covariates to include. In particular, Botosaru and Ferman (2017) highlight three cases in which, even when there is a perfect pre-treatment fit of the outcomes, a good fit on the covariates cannot be guaranteed: (i) when covariates are not a part of the DGP; (ii) if the effects of covariates are multicollinear with other observed and unobserved factors (there can be imbalances even when the covariates are relevant); and (iii) when covariates enter non-linearly in the DGP. In such situations, where there are no weights that provide a good balance in terms of both pre-treatments and covariates, the authors show that it is still possible to derive bounds for the bias of the SC estimator (provided that there are weights that deliver a perfect fit in terms of outcomes).

More recently, Kaul et al. (2021) proved an impossibility result for the SC estimator with matching on covariates. In particular, they prove that irrespective of the relevance of covariates, when all pre-treatment values are used in the definition of $\boldsymbol{z}_j$, the covariates will be irrelevant by construction,[1] i.e., there exists a solution where the weights in the matrix $\boldsymbol{V}$ related to the covariates will be zero, and the remaining weights for the pre-treatment outcomes will be one.

It is also argued that while optimizing only with respect to the outcomes, maximizing pre-treatment fit can be harmful in terms of the total bias (as mentioned before), thus it could be

[1] Note that this is one of the possible solutions, since the estimated weights in $\boldsymbol{V}$ are not unique.

beneficial if one focuses on the bias induced by the unobserved components alone. In particular, when fitting with respect to the outcomes alone, the estimated weights might have a better performance in matching the unobserved components, such that $\boldsymbol{\Gamma}'\widehat{\boldsymbol{\omega}}^{dsc}$ better approximates $\boldsymbol{\gamma}_1$.

Overall, (i) the potential impossibility of the existence of weights that can guarantee a good fit on covariates coupled with (ii) the possible better fit in terms of matching unobservables when ignoring covariates opens room for the discussion of whether one should include covariates in the optimization. Undoubtedly, one should also acknowledge that the extent to which ignoring covariates can be harmful depends on the size of T_0 and on the relative importance of the unobserved and observed components in determining the outcome.

Appendix B. Monte Carlo Simulations: Alternative Measures

In this section, we summarize the Monte Carlo results using the two alternative measures of fit - MAB and MedAB. Both measures were also used to evaluate the in-sample placebo results. Overall, we do not document any substantially different patterns.

Table B.1: Monte Carlo Results: Total MAB.

σ	γ	**SC(B) all**	**SC(B) half**	**SC(B) one**	**SC**	**SC cov. half**	**SC cov. one**	**DSC**	**DSC cov. half**	**DSC cov. one**	**SDID**	**SDID cov. half**	**SDID cov. one**
1	0	0.8533	0.8898	0.9548	0.8570	0.8708	0.9196	0.8624	0.8720	0.9283	0.9276	0.9281	0.9441
1	0.25	0.8981	0.931	0.9633	0.897	0.9164	0.9419	0.9090	0.9259	0.9526	0.9815	0.9927	1.0012
1	1	0.9244	0.9450	0.9977	0.9231	0.9299	0.9710	0.9182	0.9321	0.9605	0.9679	0.9730	0.9771
1	1.5	0.9272	0.9653	1.0430	0.9260	0.9400	0.9892	0.9223	0.9362	0.9619	1.0028	1.0151	1.0047
0.25	0	0.0546	0.0559	0.0624	0.0564	0.0561	0.0578	0.0554	0.0560	0.0584	0.0602	0.0606	0.0619
0.25	0.25	0.0642	0.0630	0.0775	0.0632	0.0645	0.0670	0.0621	0.0632	0.0663	0.0665	0.0677	0.0682
0.25	1	0.0694	0.0680	0.0889	0.0679	0.0683	0.0727	0.0625	0.0632	0.0687	0.0655	0.0656	0.0670
0.25	1.5	0.0725	0.0709	0.0935	0.0702	0.0708	0.0775	0.0655	0.0672	0.0732	0.0693	0.0702	0.0725

Table B.2: Monte Carlo Results: Total MedAB.

σ	γ	**SC(B) all**	**SC(B) half**	**SC(B) one**	**SC**	**SC cov. half**	**SC cov. one**	**DSC**	**DSC cov. half**	**DSC cov. one**	**SDID**	**SDID cov. half**	**SDID cov. one**
1	0	0.7223	0.7231	0.7954	0.7135	0.7321	0.7942	0.7085	0.7286	0.8249	0.8095	0.7633	0.8400
1	0.25	0.7596	0.7541	0.7875	0.7589	0.7878	0.8349	0.7647	0.7631	0.7951	0.8563	0.8513	0.8549
1	1	0.7789	0.8016	0.8317	0.7807	0.7820	0.8095	0.7890	0.7702	0.7906	0.7993	0.8087	0.8055
1	1.5	0.7768	0.8201	0.9114	0.7839	0.8019	0.8305	0.7890	0.7819	0.8292	0.8447	0.8697	0.8776
0.25	0	0.0467	0.0483	0.0553	0.0494	0.0483	0.0499	0.0474	0.0475	0.0493	0.0514	0.0524	0.0539
0.25	0.25	0.0534	0.0544	0.0664	0.0549	0.0551	0.058	0.0544	0.0551	0.0584	0.0587	0.0604	0.0602
0.25	1	0.0579	0.0574	0.0744	0.0564	0.0557	0.0583	0.0520	0.0535	0.0571	0.0564	0.0570	0.0565
0.25	1.5	0.0603	0.0606	0.0799	0.0595	0.0580	0.0658	0.0557	0.0564	0.0634	0.0600	0.0602	0.0625

Appendix C. Empirical Analysis: Additional Tables and Figures

Appendix C.1. Estimated v Weights

Below we summarize the estimation results for covariate weights $\boldsymbol{v}$.

- When considering 2016:Q3 as a treatment period (Table C.3) and the mean of covariates, which is the specification adopted by Born et al. (2019), in fact, the $\boldsymbol{v}$ weights assigned to the pre-treatment outcomes are close to one and for the covariates close to zero, up to a scaling factor, for the SC and DSC methods when considering all pre-treatment outcomes, while it is not the case when considering half or one pre-treatment outcome. Moreover, this pattern does not hold for the SC(B) estimator, as expected.

- Tables C.4, C.5 and C.6 show the estimated $\boldsymbol{v}$ weights for the remaining specifications without a penalty term. When considering the treatment period 2016:Q3 and the last covariates when matching, we see that the theoretical prediction of Kaul et al. (2021) does not hold for the SC estimator. However, we should point out that this theoretical result is only one of the possible solutions for the $\boldsymbol{v}$ weights since, without a penalty term, uniqueness is not guaranteed. Moreover, for this same specification, we find that the result for the DSC estimator is in accordance with Kaul et al. (2021). Surprisingly, the DSC estimator, taking into account only half of the pre-treatment periods, also assigns zero weights $\boldsymbol{v}$ to the covariates.

- The results for the remaining two specifications show that for both for the SC and the DSC estimators when considering all pre-treatment periods, the obtained $\boldsymbol{v}$ are approximately zero for almost all the covariates. Surprisingly, for the case when considering 2016:Q2 and the last covariates, the assigned $\boldsymbol{v}$ weights when using the DSC estimator and considering half of the pre-treatment periods are again zero for the covariates. Indicating that overall, when considering the last covariates, even when covariates might be relevant, the DSC does not assign a high relevance to them.

Table C.3: Estimated $\boldsymbol{v}$ weights considering for 2016:Q3 treatment period and mean of covariates.

	SC(B) all	**SC(B) half**	**SC(B) one**	**SC all**	**SC half**	**SC one**	**DSC all**	**DSC half**	**DSC one**
2014:Q1	0.0123	0.0260	NA	0.0106	0.0215	NA	0.0159	0.0266	NA
2014:Q2	0.0121	0.0271	NA	0.0106	0.0216	NA	0.0183	0.0325	NA
2014:Q3	0.0116	0.0232	NA	0.0105	0.0208	NA	0.0166	0.0249	NA
2014:Q4	0.0119	0.0250	NA	0.0108	0.0212	NA	0.0173	0.0276	NA
2015:Q1	0.0117	0.0238	NA	0.0105	0.0210	NA	0.0138	0.0229	NA
2015:Q2	0.0114	0.0224	NA	0.0108	0.0207	NA	0.0132	0.0206	NA
2015:Q3	0.0114	0.0230	NA	0.0107	0.0208	NA	0.0135	0.0225	NA
2015:Q4	0.0122	0.0244	NA	0.0115	0.0211	NA	0.0121	0.0296	NA
2016:Q1	0.0122	0.0245	NA	0.0114	0.0211	NA	0.0135	0.0294	NA
2016:Q2	0.0111	0.0219	0.1977	0.0109	0.0206	0.3842	0.0153	0.0212	0.2018
Consumption	0.0043	0.0075	0.0218	0.0003	0.0135	0.0049	0.0000	0.0003	0.1204
Investment	0.0062	0.0236	0.0039	0.0002	0.0226	0.0006	0.0003	0.0004	0.0791
Exports	0.0151	0.0290	0.1013	0.0013	0.0253	0.2981	0.0016	0.0365	0.1672
Imports	0.0121	0.0274	0.5094	0.0002	0.0244	0.2685	0.0024	0.0325	0.0318
Labor Growth	0.0138	0.0117	0.1472	0.0031	0.0184	0.0431	0.0130	0.0165	0.2734
Employment Share	0.0040	0.0139	0.0187	0.0025	0.0224	0.0006	0.0072	0.0004	0.1263

Note: Here "NA" stands for *Not applicable*, as by definition weights are not calculated for those variables. The corresponding weights for DSC and SDID methods coincide.

Table C.4: Estimated $\boldsymbol{v}$ weights considering for 2016:Q3 treatment period and last covariate.

	SC(B) all	**SC(B) half**	**SC(B) one**	**SC all**	**SC half**	**SC one**	**DSC all**	**DSC half**	**DSC one**
2014:Q1	0.0117	0.0204	NA	0.0109	0.0225	NA	0.0186	0.0330	NA
2014:Q2	0.0111	0.0207	NA	0.0109	0.0229	NA	0.0207	0.0376	NA
2014:Q3	0.0105	0.0197	NA	0.0109	0.0225	NA	0.0183	0.0303	NA
2014:Q4	0.0105	0.0202	NA	0.0109	0.0226	NA	0.0185	0.0328	NA
2015:Q1	0.0101	0.0201	NA	0.0109	0.0219	NA	0.0117	0.0272	NA
2015:Q2	0.0099	0.0199	NA	0.0109	0.0217	NA	0.0124	0.0257	NA
2015:Q3	0.0100	0.0202	NA	0.0109	0.0216	NA	0.0120	0.0256	NA
2015:Q4	0.0102	0.0206	NA	0.0109	0.0221	NA	0.0136	0.0281	NA
2016:Q1	0.0103	0.0206	NA	0.0109	0.0218	NA	0.0128	0.0271	NA
2016:Q2	0.0098	0.0201	0.0761	0.0109	0.0213	0.1899	0.0115	0.0237	0.3179
Consumption	0.0162	0.0058	0.0133	0.0109	0.0017	0.0652	0.0032	0.0005	0.1436
Investment	0.0014	0.0015	0.0015	0.0109	0.0004	0.0037	0.0002	0.0007	0.0000
Exports	0.0109	0.0249	0.1565	0.0109	0.0219	0.0201	0.0020	0.0068	0.0002
Imports	0.0124	0.0193	0.1177	0.0109	0.0217	0.1255	0.0022	0.0084	0.1793
Labor Productivy Growth	0.1427	0.1103	0.6312	0.0087	0.0023	0.1132	0.0002	0.0080	0.0246
Employment Share	0.0074	0.0037	0.0038	0.0109	0.0003	0.4824	0.0125	0.0002	0.3343

Table C.5: Estimated $\boldsymbol{v}$ weights considering for 2016:Q2 treatment period and mean of covariates.

	SC(B) all	**SC(B) half**	**SC(B) one**	**SC all**	**SC half**	**SC one**	**DSC all**	**DSC half**	**DSC one**
2014:Q1	0.0113	0.0216	NA	0.0102	0.0207	NA	0.0140	0.0242	NA
2014:Q2	0.0122	0.0262	NA	0.0096	0.0206	NA	0.0162	0.0267	NA
2014:Q3	0.0121	0.0274	NA	0.0098	0.0204	NA	0.0159	0.0338	NA
2014:Q4	0.0117	0.0234	NA	0.0096	0.0207	NA	0.0146	0.0220	NA
2015:Q1	0.0120	0.0252	NA	0.0097	0.0205	NA	0.0152	0.0251	NA
2015:Q2	0.0117	0.0240	NA	0.0103	0.0208	NA	0.0112	0.0235	NA
2015:Q3	0.0114	0.0226	NA	0.0106	0.0207	NA	0.0113	0.0215	NA
2015:Q4	0.0114	0.0232	NA	0.0106	0.0207	NA	0.0112	0.0246	NA
2016:Q1	0.0121	0.0246	NA	0.0112	0.0208	NA	0.0120	0.0373	NA
2016:Q2	0.0121	0.0247	0.1917	0.0112	0.0208	0.1315	0.0117	0.0373	0.0011
Consumption	0.0044	0.0081	0.0227	0.0000	0.0136	0.1639	0.0047	0.0000	0.0063
Investment	0.0061	0.0250	0.0054	0.0000	0.0232	0.0425	0.0001	0.0001	0.0137
Exports	0.0145	0.0294	0.3795	0.0012	0.0237	0.1958	0.0111	0.0481	0.4975
Imports	0.0125	0.0277	0.2110	0.0027	0.0230	0.2009	0.0093	0.0330	0.4536
Labor Productivy Growth	0.0112	0.0151	0.1620	0.0048	0.0214	0.2641	0.0026	0.0100	0.0217
Employment Share	0.0043	0.0147	0.0276	0.0002	0.0222	0.0012	0.0070	0.0000	0.0061

Table C.6: Estimated $\boldsymbol{v}$ weights considering for 2016:Q2 treatment period and last covariate.

	SC(B) all	**SC(B) half**	**SC(B) one**	**SC all**	**SC half**	**SC one**	**DSC all**	**DSC half**	**DSC one**
2014:Q1	0.0117	0.0233	NA	0.0113	0.0215	NA	0.0147	0.0232	NA
2014:Q2	0.0114	0.0294	NA	0.0115	0.0227	NA	0.0170	0.0301	NA
2014:Q3	0.0115	0.0312	NA	0.0116	0.0225	NA	0.0191	0.0351	NA
2014:Q4	0.0113	0.0285	NA	0.0115	0.0218	NA	0.0177	0.0280	NA
2015:Q1	0.0113	0.0298	NA	0.0119	0.0226	NA	0.0180	0.0315	NA
2015:Q2	0.0113	0.0240	NA	0.0115	0.0228	NA	0.0094	0.0230	NA
2015:Q3	0.0111	0.0225	NA	0.0116	0.0230	NA	0.0109	0.0243	NA
2015:Q4	0.0113	0.0228	NA	0.0115	0.0235	NA	0.0107	0.0238	NA
2016:Q1	0.0112	0.0219	NA	0.0121	0.0260	NA	0.0125	0.0231	NA
2016:Q2	0.0125	0.0223	0.0543	0.0120	0.0254	0.2745	0.0119	0.0236	0.3826
Consumption	0.0129	0.0048	0.0117	0.0124	0.0012	0.1248	0.0035	0.0003	0.0449
Investment	0.0019	0.0008	0.0012	0.0013	0.0000	0.0003	0.0016	0.0011	0.0007
Exports	0.0167	0.0224	0.2427	0.0038	0.0215	0.0504	0.0018	0.0062	0.2788
Imports	0.0170	0.0213	0.1482	0.0027	0.0221	0.1434	0.0005	0.0006	0.1418
Labor Productivy Growth	0.0806	0.1000	0.5365	0.0000	0.0103	0.0164	0.0023	0.0033	0.0066
Employment Share	0.0271	0.0018	0.0054	0.0043	0.0457	0.3902	0.0102	0.0001	0.1447

Appendix C.2. Estimated ω Weights

Tables C.7, C.8, and C.9 summarize the estimated $\boldsymbol{\omega}$ weights for each of the specifications without a penalty term.

- For the majority of the estimators and specifications, the highest weight is assigned to the United States (US). Some exceptions are: for the SC and DSC estimators without covariates, independently of the specification, the highest weight is always assigned to Hungary; for the SC estimator with all pre-treatment periods and mean of covariates - as expected since covariates do not play a role - the highest weight is also assigned to Hungary; the SC estimator with one covariate and including last covariates always assigns higher weights to New Zealand. Less systematically across all estimators, other countries that received the highest weights in a few cases are Canada and Portugal.

- One finding that holds across most of the specifications (except for the treatment date 2016:Q2 when taking into account the last covariates) is that the weights assigned to the US are always higher for the SC(B) estimators and the SC or DSC estimators with matching on covariates. Moreover, the weights assigned to the US are always higher when taking into account the mean of the covariates, as opposed to consider the last covariates.

Table C.7: Estimated ω weights considering 2016:Q3 as treatment period and mean of covariates.

	SC(B) all	**SC(B) half**	**SC(B) one**	**SC no cov**	**SC all**	**SC half**	**SC one**	**DSC no cov**	**DSC all**	**DSC half**	**DSC one**
Australia	0.0000	0.0000	0.0000	0.0052	0.0005	0.0000	0.0002	0.0075	0.0000	0.0000	0.0000
Austria	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0005	0.0000	0.0000	0.0000	0.0000
Belgium	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0036	0.0000	0.0000	0.0000	0.0001
Canada	0.0000	0.0001	0.0000	0.1612	0.1734	0.0000	0.0004	0.1916	0.2103	0.0705	0.0001
Finland	0.0000	0.0000	0.0000	0.0021	0.0002	0.0000	0.0003	0.0031	0.0103	0.0000	0.0000
France	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002	0.0000	0.0000	0.0000	0.0000
Germany	0.0458	0.1145	0.0000	0.0026	0.0000	0.1012	0.0004	0.0000	0.0000	0.0000	0.0379
Hungary	0.1078	0.0001	0.0000	0.2186	0.2262	0.0000	0.0011	0.2311	0.2258	0.1697	0.0000
Iceland	0.0089	0.0682	0.0000	0.0000	0.0076	0.0740	0.0006	0.0000	0.0000	0.0350	0.0569
Ireland	0.0114	0.0001	0.0000	0.0543	0.0462	0.0000	0.0002	0.0503	0.0469	0.0305	0.0000
Italy	0.1744	0.1631	0.1667	0.0353	0.0013	0.1673	0.2098	0.0334	0.0373	0.0471	0.1773
Japan	0.0000	0.0000	0.0000	0.1773	0.1821	0.0000	0.0000	0.1842	0.1838	0.1884	0.0000
Korea	0.0000	0.0000	0.0000	0.0030	0.0000	0.0000	0.0002	0.0000	0.0000	0.0000	0.0000
Luxembourg	0.0000	0.0483	0.0310	0.0000	0.0000	0.0371	0.0535	0.0000	0.0000	0.0000	0.0382
Netherlands	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0013	0.0000	0.0001	0.0000	0.0000
New Zealand	0.1432	0.0001	0.0586	0.0000	0.0000	0.0000	0.1183	0.0000	0.0144	0.0001	0.0001
Norway	0.0001	0.0000	0.0000	0.1256	0.1133	0.0000	0.0001	0.1208	0.1042	0.0000	0.0000
Portugal	0.0000	0.0001	0.0000	0.0123	0.0407	0.0000	0.0024	0.0037	0.0011	0.0000	0.0000
Slovak Republic	0.0000	0.0000	0.0000	0.0031	0.0000	0.0000	0.0005	0.0000	0.0000	0.0000	0.0000
Spain	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000
Sweden	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002	0.0000	0.0000	0.0000	0.0000
Switzerland	0.0000	0.0000	0.1283	0.0000	0.0000	0.0000	0.0169	0.0000	0.0000	0.0000	0.0001
United States	0.5083	0.6052	0.6153	0.1994	0.2083	0.6203	0.5893	0.1740	0.1655	0.4587	0.6891

Table C.8: Estimated ω weights considering 2016:Q3 as treatment period and last covariates.

	SC(B) all	**SC(B) half**	**SC(B) one**	**SC no cov**	**SC all**	**SC half**	**SC one**	**DSC no cov**	**DSC all**	**DSC half**	**DSC one**
Australia	0.0001	0.0000	0.0000	0.0052	0.0509	0.0001	0.0004	0.0075	0.0000	0.0000	0.0000
Austria	0.0001	0.0000	0.0000	0.0000	0.0000	0.0002	0.0005	0.0000	0.0000	0.0000	0.0004
Belgium	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0004	0.0000	0.0000	0.0000	0.0003
Canada	0.0343	0.0000	0.0000	0.1612	0.1167	0.0050	0.0007	0.1916	0.1909	0.1827	0.0003
Finland	0.0000	0.0000	0.0000	0.0021	0.0000	0.0008	0.0004	0.0031	0.0073	0.0000	0.0004
France	0.0000	0.0000	0.0000	0.0000	0.0000	0.0003	0.0002	0.0000	0.0000	0.0000	0.0002
Germany	0.0015	0.0000	0.0000	0.0026	0.0016	0.0004	0.0014	0.0000	0.0001	0.0000	0.0005
Hungary	0.0454	0.0639	0.0000	0.2186	0.1022	0.0650	0.0005	0.2311	0.2193	0.1825	0.0003
Iceland	0.0006	0.0113	0.0000	0.0000	0.0009	0.0162	0.0070	0.0000	0.0023	0.0105	0.0003
Ireland	0.0000	0.0000	0.0000	0.0543	0.0000	0.0329	0.0001	0.0503	0.0452	0.0504	0.0002
Italy	0.1576	0.1967	0.0282	0.0353	0.0597	0.1582	0.0001	0.0334	0.0000	0.0785	0.0002
Japan	0.0066	0.0271	0.0164	0.1773	0.0812	0.0909	0.0995	0.1842	0.2182	0.1819	0.0975
Korea	0.0000	0.0000	0.0000	0.0030	0.0000	0.0001	0.0002	0.0000	0.0000	0.0000	0.0002
Luxembourg	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0003	0.0000	0.0000	0.0000	0.0002
Netherlands	0.0001	0.0000	0.0000	0.0000	0.0001	0.0002	0.0005	0.0000	0.0001	0.0000	0.0003
New Zealand	0.2804	0.2972	0.3139	0.0000	0.0878	0.1303	0.3344	0.0000	0.0299	0.0470	0.3172
Norway	0.0001	0.0000	0.0000	0.1256	0.0002	0.0001	0.0003	0.1208	0.0000	0.0000	0.0003
Portugal	0.0798	0.0012	0.2876	0.0123	0.1116	0.0001	0.2191	0.0037	0.0430	0.0000	0.2139
Slovak Republic	0.0000	0.0000	0.0000	0.0031	0.0001	0.0002	0.0004	0.0000	0.0000	0.0000	0.0002
Spain	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0002	0.0000	0.0001	0.0000	0.0002
Sweden	0.0001	0.0000	0.0000	0.0000	0.0002	0.0004	0.0003	0.0000	0.0000	0.0000	0.0002
Switzerland	0.0007	0.0000	0.0000	0.0000	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0042
United States	0.3925	0.4025	0.3538	0.1994	0.3865	0.4980	0.3331	0.1740	0.2436	0.2664	0.3623

Table C.9: Estimated ω weights considering 2016:Q2 as treatment period and mean of covariates.

	SC(B) all	**SC(B) half**	**SC(B) one**	**SC no cov**	**SC all**	**SC half**	**SC one**	**DSC no cov**	**DSC all**	**DSC half**	**DSC one**
Australia	0.0000	0.0000	0.0000	0.0043	0.0005	0.0000	0.0000	0.0007	0.0000	0.0000	0.0000
Austria	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Belgium	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000
Canada	0.0000	0.0000	0.0002	0.1526	0.1812	0.0000	0.0001	0.1868	0.0000	0.0000	0.0000
Finland	0.0000	0.0000	0.0000	0.0025	0.0013	0.0000	0.0000	0.0000	0.0002	0.0000	0.0000
France	0.0000	0.0000	0.0000	0.0041	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Germany	0.0530	0.1154	0.0000	0.0059	0.0000	0.0990	0.0003	0.0044	0.0001	0.0000	0.0001
Hungary	0.1189	0.0000	0.0000	0.2208	0.2194	0.0000	0.0001	0.2330	0.2027	0.2282	0.0000
Iceland	0.0118	0.0654	0.0001	0.0000	0.0003	0.0732	0.0001	0.0002	0.0001	0.0016	0.0359
Ireland	0.0077	0.0000	0.0000	0.0553	0.0564	0.0000	0.0001	0.0501	0.0394	0.0275	0.0000
Italy	0.1612	0.1614	0.1529	0.0285	0.0347	0.1688	0.2156	0.0294	0.0534	0.0196	0.2065
Japan	0.0000	0.0000	0.0000	0.1803	0.1765	0.0000	0.0000	0.1823	0.1741	0.1823	0.0000
Korea	0.0000	0.0000	0.0000	0.0041	0.0000	0.0000	0.0000	0.0024	0.0000	0.0000	0.0000
Luxembourg	0.0004	0.0490	0.0382	0.0000	0.0000	0.0370	0.0586	0.0000	0.0000	0.0000	0.0713
Netherlands	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0001
New Zealand	0.1207	0.0000	0.0114	0.0000	0.0000	0.0000	0.0001	0.0000	0.0673	0.0000	0.0000
Norway	0.0001	0.0000	0.0000	0.1249	0.1323	0.0000	0.0000	0.1228	0.0000	0.0000	0.0000
Portugal	0.0000	0.0000	0.0000	0.0135	0.0078	0.0000	0.0002	0.0058	0.0086	0.0000	0.0000
Slovak Republic	0.0000	0.0000	0.0000	0.0021	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Spain	0.0000	0.0000	0.0001	0.0000	0.0005	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000
Sweden	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Switzerland	0.0000	0.0000	0.1283	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000
United States	0.5259	0.6088	0.6689	0.2010	0.1890	0.6221	0.7242	0.1823	0.4540	0.5408	0.6860

Table C.10: Estimated ω weights considering 2016:Q2 as treatment period and last covariates.

	SC(B) all	**SC(B) half**	**SC(B) one**	**SC no cov**	**SC all**	**SC half**	**SC one**	**DSC no cov**	**DSC all**	**DSC half**	**DSC one**
Australia	0.0000	0.0000	0.2509	0.0043	0.0013	0.0000	0.0000	0.0007	0.0000	0.0000	0.0000
Austria	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0001
Belgium	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0001
Canada	0.3296	0.3242	0.0000	0.1526	0.0108	0.1970	0.0000	0.1868	0.1893	0.2706	0.0001
Finland	0.0000	0.0000	0.0000	0.0025	0.0001	0.0459	0.0000	0.0000	0.0014	0.0000	0.0001
France	0.0000	0.0000	0.0000	0.0041	0.0000	0.0004	0.0000	0.0000	0.0000	0.0001	0.0001
Germany	0.0000	0.0001	0.0000	0.0059	0.0005	0.0001	0.0001	0.0044	0.0001	0.0000	0.0003
Hungary	0.0017	0.0001	0.0000	0.2208	0.2239	0.0001	0.0000	0.2330	0.2173	0.1550	0.0001
Iceland	0.0244	0.0002	0.0519	0.0000	0.0001	0.0650	0.0008	0.0002	0.0055	0.0460	0.0266
Ireland	0.0000	0.0360	0.0000	0.0553	0.0000	0.0678	0.0000	0.0501	0.0461	0.0530	0.0000
Italy	0.0256	0.0001	0.0000	0.0285	0.0009	0.0005	0.0001	0.0294	0.0000	0.0679	0.0001
Japan	0.0940	0.1406	0.0249	0.1803	0.1334	0.2909	0.1027	0.1823	0.2187	0.2145	0.0737
Korea	0.0000	0.0000	0.0000	0.0041	0.0000	0.0000	0.0000	0.0024	0.0000	0.0000	0.0000
Luxembourg	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Netherlands	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0001	0.0000	0.0001
New Zealand	0.1411	0.0000	0.0000	0.0000	0.0314	0.0000	0.3611	0.0000	0.0321	0.0000	0.3185
Norway	0.0000	0.0000	0.0000	0.1249	0.0001	0.0012	0.0000	0.1228	0.0001	0.0002	0.0001
Portugal	0.1647	0.1650	0.3801	0.0135	0.0467	0.0516	0.2110	0.0058	0.0469	0.0000	0.2365
Slovak Republic	0.0000	0.0000	0.0000	0.0021	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001
Spain	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002	0.0000	0.0000	0.0001	0.0000	0.0001
Sweden	0.0001	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001
Switzerland	0.0005	0.0000	0.0000	0.0000	0.0010	0.0000	0.0016	0.0000	0.0000	0.0000	0.0184
United States	0.2180	0.3335	0.2921	0.2010	0.5496	0.2787	0.3224	0.1823	0.2422	0.1927	0.3247

Appendix C.3. Estimated Counterfactuals for the Treatment Period 2016:Q3

Figures C.1 and C.2 show the additional counterfactual graphs when we define 2016:Q3 as the treatment period.

- Note that for all remaining specifications, the methods generate counterfactual trends that are able to replicate the GDP trend of the UK prior to the treatment date. It is noteworthy that all the figures also show that right after the financial crises, all methods overestimated the GDP of the UK, as expected, but in the periods prior to Brexit, the counterfactuals and the actual GDP are more aligned.

- Moreover, we see that the counterfactuals obtained with the DSC and the SDID methods in both figures are parallel, indicating that, indeed, the difference between such methods is driven only by the difference in the bias adjustment term that is constant over time. Finally, for the case considering all pre-treatment periods, the SC(B) taking into account the mean of or the last covariates point out to a significantly smaller treatment effect, and the SC considering the last covariates underestimates consistently the GDP of the UK in the periods prior to treatment. For the case with one pre-treatment period, it is also clear that the SC(B) mainly taking into account the mean of covariates, underestimates the GDP of the UK.

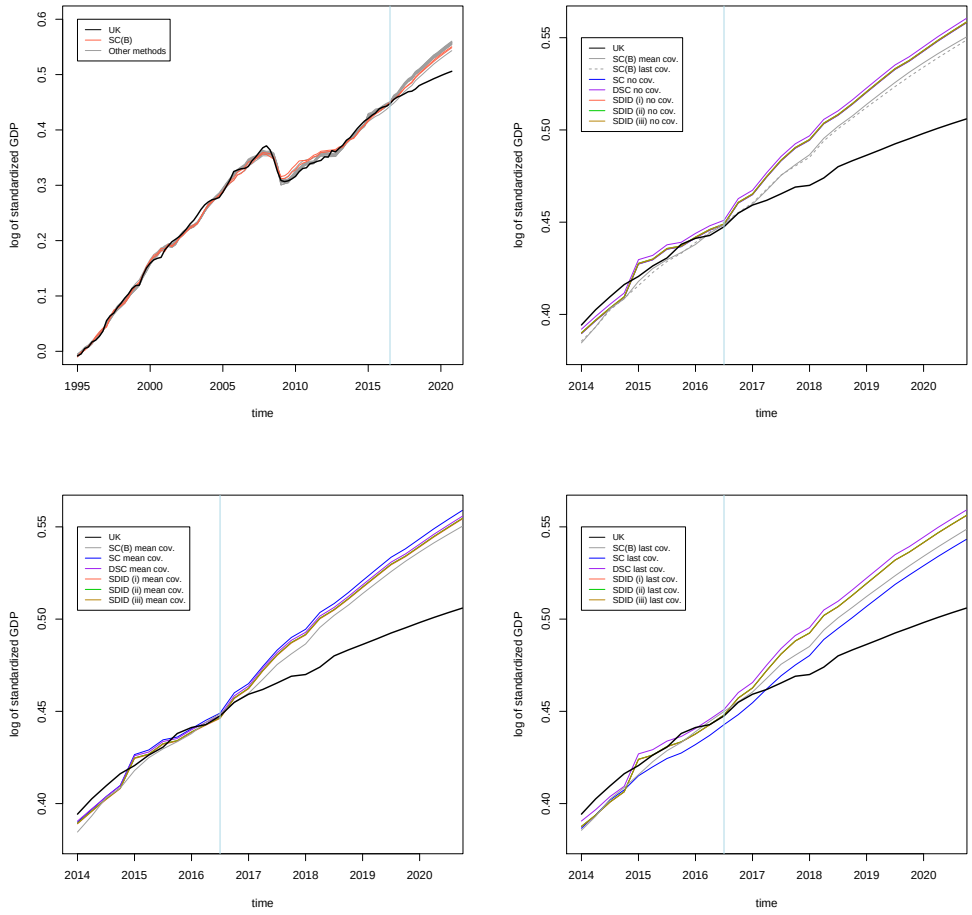


Figure C.1: Estimated counterfactuals for treatment period 2016:Q3 and all pre-treatment outcomes.

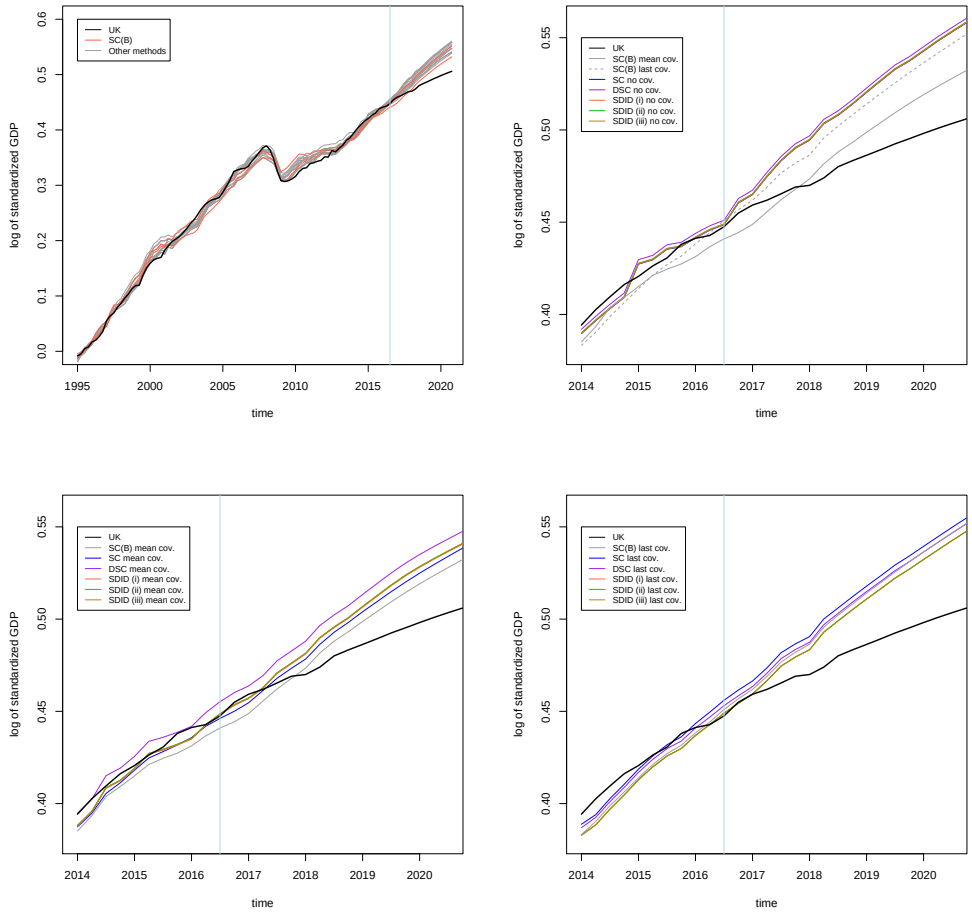


Figure C.2: Estimated counterfactuals for treatment period 2016:Q3 and one pre-treatment outcome.

Appendix C.4. Estimated Counterfactuals for Each of the Placebos over Time

Similarly to Born et al. (2019) and to what is typically done in practice, we plot the counterfactuals obtained for each placebo treatment period to inspect whether there is a causal relationship between Brexit and the estimated effects. Intuitively, a causal relationship is attributed as long as similar magnitudes in the change of the trajectory of the counterfactual estimates are not observed for the placebo treatment periods.

- The trajectories in Figure C.3 show that in this application, this is the case when limiting our attention to the specification with only half pre-treatment periods (while matching on covariates).
- Similarly, Figures C.4 and C.5 show that, for the remaining specifications, the counterfactuals obtained with the in-sample placebo analysis are all parallel to the counterfactual estimated in the treatment period.

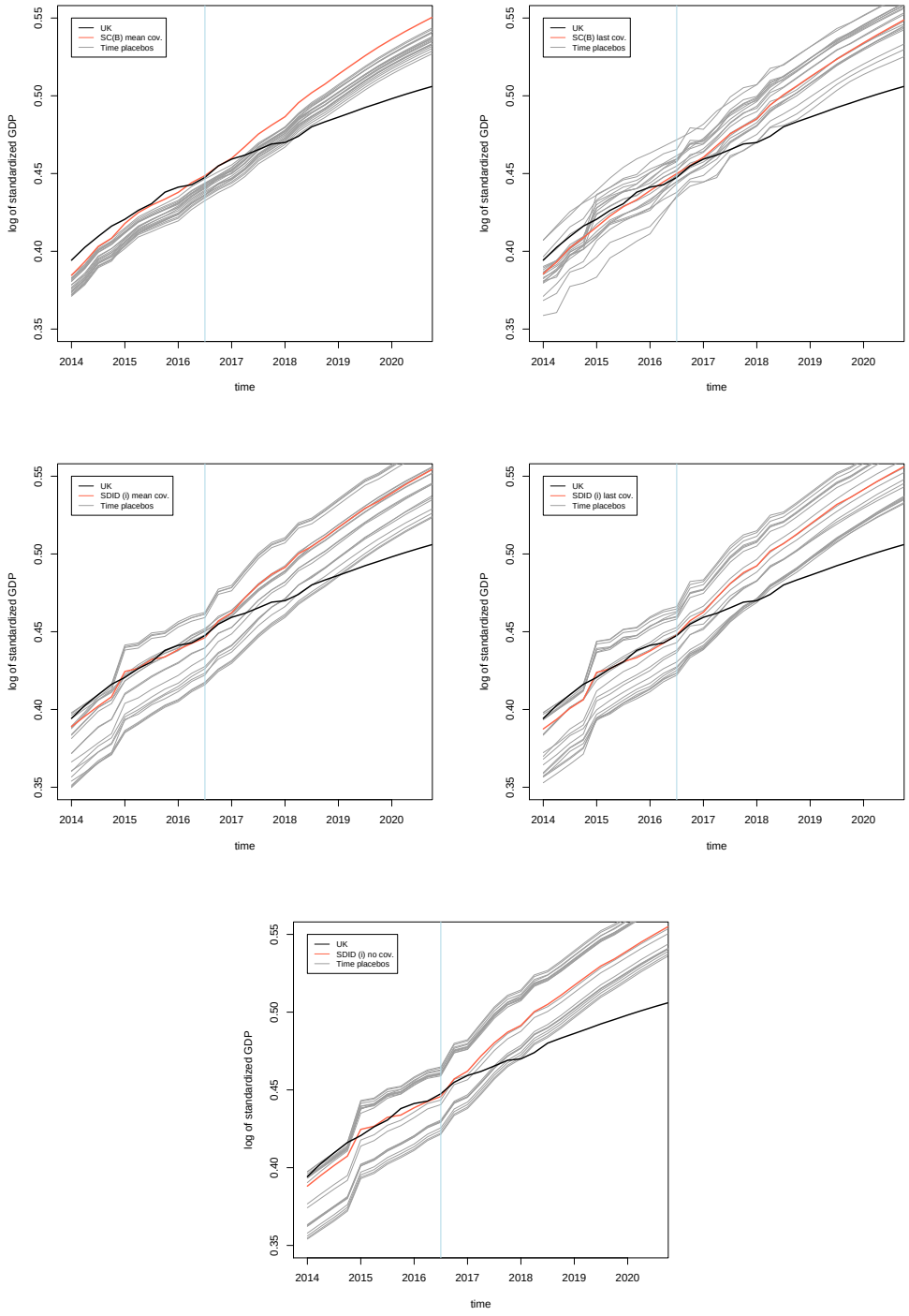


Figure C.4: Counterfactuals for each of the placebos with treatment dates over the period 2010:Q1-2014:Q4, considering all the pre-treatment periods.

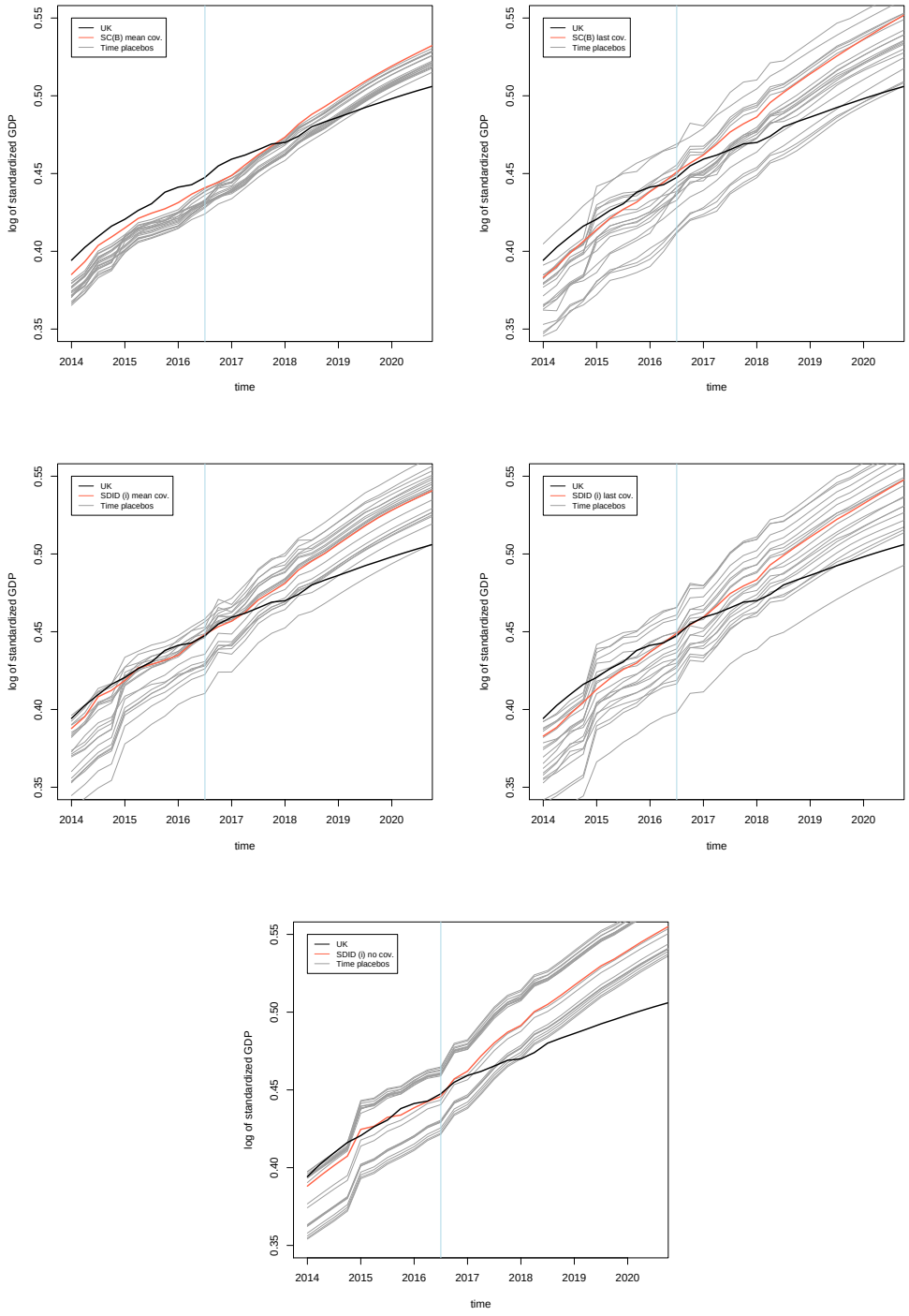


Figure C.5: Counterfactuals for each of the placebos with treatment dates over the period 2010:Q1-2014:Q4, considering one of the pre-treatment periods.

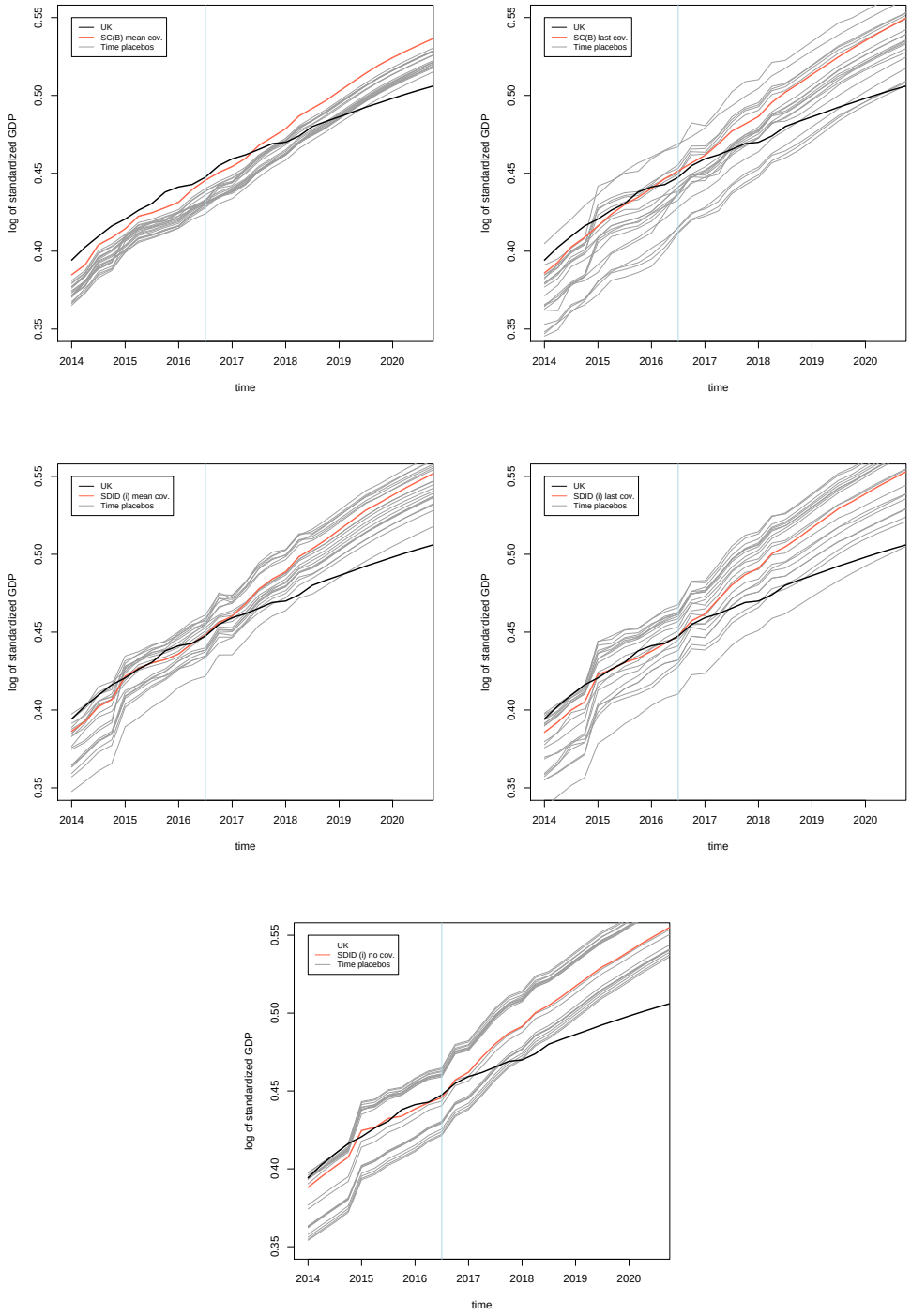
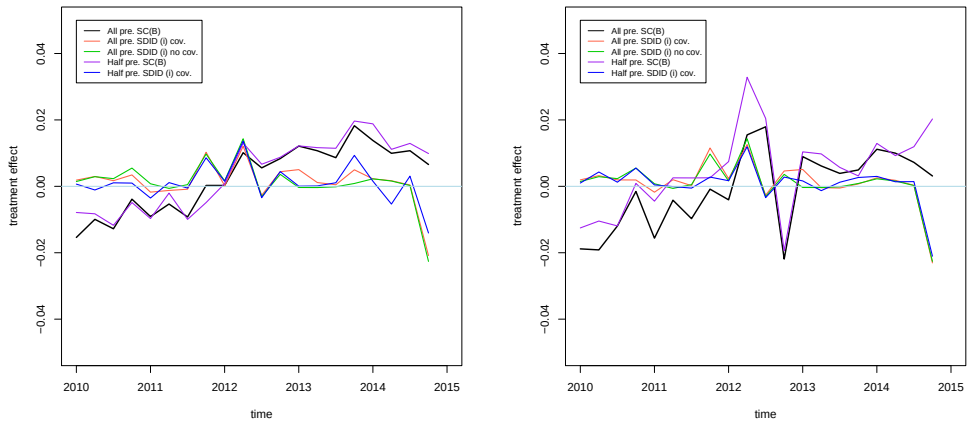


Figure C.3: Counterfactuals for each of the placebos with treatment dates over the period 2010:Q1-2014:Q4, considering half of the pre-treatment periods.



(a) Matching on the mean of covariates.

(b) Matching on the last covariate.

Figure C.6: Estimated treatment effect over the period 2010:Q1-2014:Q4.

Appendix D. Empirical Analysis: Estimation with Ridge Penalization

Appendix D.1. Estimated Treatment Effects

Tables D.11, D.12, and D.13 show the estimated treatment effects for all methods and specifications when taking into account a penalty regularization term as proposed by Arkhangelsky et al. (2021).

- When considering all pre-treatment periods (Table D.11), we see that for the specification considered by Born et al. (2019), which is setting the treatment period to be 2016:Q3 (and considering the mean of covariates when matching on covariates), the method SC(B) delivers a slightly smaller estimate of the treatment effect when compared to the original paper that does not take into account a penalty term (2.38% versus 2.43%). Across the different specifications for the treatment date and how covariates are taken into account, the results obtained with the SC(B) method are mixed in terms of whether we obtain higher or smaller effects when taking into account the penalty term.

- Regarding the remaining methods, when considering covariates, the effects estimated with a penalty term are always smaller than without; and when not considering covariates the effects are generally bigger - except for the SDID method, but the difference with respect to the estimates without a penalty is negligible.

- When comparing across methods (with a penalty term included) based on all pre-treatment periods, we see that (i) for any specification, the other methods without covariates point out to a bigger estimated effect than 2.38%; and (ii) also comparing with the estimates delivered by SC(B) for a given specification, the other methods without covariates always point out to a higher effect. The same conclusions generally apply to the case when we consider half of the pre-treatment periods, in Table D.12, with the exception being that the SC methods with covariates point out to a smaller effect for a given specification with penalty term compared to the SC(B) method.

- On the other hand, the conclusions obtained when we consider only one pre-treatment period (Table D.13) are very mixed in terms of comparing whether the effects when considering a penalty term are bigger or smaller, without a clear pattern. However, it still holds that all other specifications and methods with penalty point out to a bigger

effect compared to the SC(B) estimator under the specification of Born et al. (2019) when also considering such a regularization, except for the DSC with covariates estimator.

Table D.11: The estimated percentage differences in GDP between the counterfactuals and the UK including all pre-treatment periods as predictors, with penalty.

	SC(B)	**SC**	**SC cov.**	**DSC**	**DSC cov.**	**SDID (i)**	**SDID cov. (i)**	**SDID (ii)**	**SDID cov. (ii)**	**SDID (iii)**	**SDID cov. (iii)**
Mean of covariates - Treatment period 2016:Q2											
2018:Q4	2.42	3.19	2.39	3.19	2.13	3.16	2.71	2.85	2.55	2.94	2.55
2019:Q4	3.63	4.28	3.60	4.29	3.24	4.26	3.81	3.92	3.65	4.04	3.65
Mean of covariates - Treatment period 2016:Q3											
2018:Q4	2.38	3.09	2.31	3.09	2.14	2.64	2.16	2.53	2.17	2.66	2.17
2019:Q4	3.60	4.17	3.52	4.18	3.25	3.74	3.26	3.59	3.28	3.76	3.28
Last covariates - Treatment period 2016:Q2											
2018:Q4	0.52	3.19	0.79	3.19	1.97	3.16	2.99	2.85	2.85	2.94	2.85
2019:Q4	1.68	4.28	1.93	4.29	3.06	4.26	4.08	3.92	3.94	4.04	3.94
Last covariates - Treatment period 2016:Q3											
2018:Q4	1.50	3.09	1.53	3.09	1.80	2.64	2.37	2.53	2.42	2.66	2.42
2019:Q4	2.61	4.17	2.66	4.18	2.92	3.74	3.49	3.59	3.54	3.76	3.54

Table D.12: The estimated percentage differences in GDP between the counterfactuals and the UK including half pre-treatment periods as predictors, with penalty.

	SC(B)	**SC**	**SC cov.**	**DSC**	**DSC cov.**	**SDID (i)**	**SDID cov. (i)**	**SDID (ii)**	**SDID cov. (ii)**	**SDID (iii)**	**SDID cov. (iii)**
Mean of covariates - Treatment period 2016:Q2											
2018:Q4	1.46	3.19	1.32	3.19	1.99	3.16	2.73	2.85	2.65	2.94	2.65
2019:Q4	2.64	4.28	2.45	4.29	3.12	4.26	3.87	3.92	3.79	4.04	3.79
Mean of covariates - Treatment period 2016:Q3											
2018:Q4	1.46	3.09	1.31	3.09	2.00	2.64	2.05	2.53	2.08	2.66	2.08
2019:Q4	2.65	4.17	2.44	4.18	3.16	3.74	3.21	3.59	3.24	3.76	3.24
Last covariates - Treatment period 2016:Q2											
2018:Q4	0.27	3.19	0.58	3.19	2.02	3.16	2.87	2.85	2.72	2.94	2.72
2019:Q4	1.40	4.28	1.66	4.29	3.06	4.26	3.92	3.92	3.77	4.04	3.77
Last covariates - Treatment period 2016:Q3											
2018:Q4	1.54	3.09	1.84	3.09	1.68	2.64	2.10	2.53	2.14	2.66	2.14
2019:Q4	2.62	4.17	2.92	4.18	2.75	3.74	3.17	3.59	3.22	3.76	3.22

Table D.13: The estimated percentage differences in GDP between the counterfactuals and the UK including one pre-treatment periods as predictors, with penalty.

	SC(B)	**SC**	**SC cov.**	**DSC**	**DSC cov.**	**SDID (i)**	**SDID cov. (i)**	**SDID (ii)**	**SDID cov. (ii)**	**SDID (iii)**	**SDID cov. (iii)**
Mean of covariates - Treatment period 2016:Q2											
2018:Q4	1.28	3.19	1.63	3.19	1.42	3.16	2.44	2.85	2.42	2.94	2.42
2019:Q4	2.24	4.28	2.67	4.29	2.53	4.26	3.55	3.92	3.54	4.04	3.54
Mean of covariates - Treatment period 2016:Q3											
2018:Q4	0.84	3.09	1.17	3.09	1.38	2.64	1.73	2.53	1.78	2.66	1.78
2019:Q4	1.76	4.17	2.18	4.18	2.49	3.74	2.84	3.59	2.89	3.76	2.89
Last covariates - Treatment period 2016:Q2											
2018:Q4	2.31	3.19	2.79	3.19	2.89	3.16	3.14	2.85	2.84	2.94	2.84
2019:Q4	3.55	4.28	3.81	4.29	3.98	4.26	4.23	3.92	3.93	4.04	3.93
Last covariates - Treatment period 2016:Q3											
2018:Q4	2.69	3.09	2.15	3.09	2.09	2.64	2.08	2.53	2.13	2.66	2.13
2019:Q4	3.79	4.17	3.22	4.18	3.15	3.74	3.13	3.59	3.19	3.76	3.19

Appendix D.2. Estimated v Weights

Tables D.14 - D.17 show the estimated $\boldsymbol{v}$ weights for the different methods and under different specifications with regularization.

- As expected, since the inclusion of the latter leads to a different inner and outer optimization in the cases with matching on covariates, the theoretical prediction of Kaul et al. (2021) does not hold here, such that the covariates are not necessarily irrelevant even when considering all pre-treatment periods.

- Specifically, we have that across the specifications, all the methods that take into account covariates and all pre-treatment periods essentially assign the same weights to the predictors (outcomes in the pre-treatment periods and the covariates).

Table D.14: Estimated $\boldsymbol{v}$ weights considering 2016:Q3 as treatment period, mean of covariates and a ridge penalty.

	SC(B) all	**SC(B) half**	**SC(B) one**	**SC all**	**SC half**	**SC one**	**DSC all**	**DSC half**	**DSC one**
2014:Q1	0.0109	0.0205	NA	0.0109	0.0211	NA	0.0109	0.0217	NA
2014:Q2	0.0109	0.0207	NA	0.0109	0.0230	NA	0.0109	0.0231	NA
2014:Q3	0.0109	0.0205	NA	0.0109	0.0205	NA	0.0109	0.0214	NA
2014:Q4	0.0109	0.0206	NA	0.0109	0.0208	NA	0.0109	0.0215	NA
2015:Q1	0.0109	0.0206	NA	0.0109	0.0207	NA	0.0109	0.0213	NA
2015:Q2	0.0109	0.0206	NA	0.0109	0.0204	NA	0.0109	0.0206	NA
2015:Q3	0.0109	0.0205	NA	0.0109	0.0204	NA	0.0109	0.0212	NA
2015:Q4	0.0109	0.0206	NA	0.0109	0.0207	NA	0.0109	0.0215	NA
2016:Q1	0.0109	0.0206	NA	0.0109	0.0206	NA	0.0109	0.0215	NA
2016:Q2	0.0109	0.0207	0.1858	0.0109	0.0203	0.4597	0.0109	0.0205	0.3031
Consumption	0.0104	0.0112	0.0480	0.0109	0.0249	0.0174	0.0109	0.0195	0.1323
Investment	0.0099	0.0239	0.0099	0.0109	0.0218	0.0104	0.0087	0.0123	0.0860
Exports	0.0110	0.0216	0.5006	0.0109	0.0211	0.2666	0.0109	0.0213	0.0921
Imports	0.0110	0.0214	0.0000	0.0109	0.0209	0.0376	0.0109	0.0214	0.0553
Labor Productivity Growth	0.0110	0.0205	0.2040	0.0109	0.0194	0.2081	0.0109	0.0192	0.2237
Employment Share	0.0104	0.0193	0.0516	0.0087	0.0176	0.0001	0.0109	0.0211	0.1075

Table D.15: Estimated $\boldsymbol{v}$ weights considering 2016:Q3 as treatment period, last covariates and a ridge penalty.

	SC(B) all	**SC(B) half**	**SC(B) one**	**SC all**	**SC half**	**SC one**	**DSC all**	**DSC half**	**DSC one**
2014:Q1	0.0109	0.0206	NA	0.0109	0.0210	NA	0.0109	0.0219	NA
2014:Q2	0.0109	0.0204	NA	0.0109	0.0210	NA	0.0110	0.0223	NA
2014:Q3	0.0109	0.0170	NA	0.0109	0.0210	NA	0.0109	0.0218	NA
2014:Q4	0.0109	0.0229	NA	0.0109	0.0210	NA	0.0110	0.0219	NA
2015:Q1	0.0109	0.0206	NA	0.0109	0.0217	NA	0.0109	0.0217	NA
2015:Q2	0.0109	0.0211	NA	0.0109	0.0214	NA	0.0110	0.0217	NA
2015:Q3	0.0109	0.0208	NA	0.0109	0.0215	NA	0.0109	0.0216	NA
2015:Q4	0.0109	0.0205	NA	0.0109	0.0210	NA	0.0109	0.0232	NA
2016:Q1	0.0109	0.0205	NA	0.0109	0.0211	NA	0.0109	0.0217	NA
2016:Q2	0.0109	0.0189	0.0590	0.0109	0.0221	0.5680	0.0109	0.0209	0.2763
Consumption	0.0109	0.0207	0.0868	0.0109	0.0196	0.0000	0.0108	0.0210	0.0000
Investment	0.0109	0.0165	0.0018	0.0109	0.0104	0.0047	0.0099	0.0176	0.0054
Exports	0.0109	0.0208	0.1152	0.0109	0.0204	0.2239	0.0108	0.0204	0.3949
Imports	0.0109	0.0216	0.1112	0.0109	0.0210	0.1509	0.0109	0.0204	0.3030
Labor Productivity Growth	0.0119	0.0249	0.6183	0.0087	0.0204	0.0048	0.0105	0.0193	0.0067
Employment Share	0.0109	0.0202	0.0077	0.0109	0.0208	0.0475	0.0111	0.0182	0.0137

Table D.16: Estimated $\boldsymbol{v}$ weights considering 2016:Q2 as treatment period, mean of covariates and a ridge penalty.

	SC(B) all	**SC(B) half**	**SC(B) one**	**SC all**	**SC half**	**SC one**	**DSC all**	**DSC half**	**DSC one**
2014:Q1	0.0110	0.0220	NA	0.0111	0.0205	NA	0.0110	0.0210	NA
2014:Q2	0.0110	0.0213	NA	0.0110	0.0216	NA	0.0110	0.0208	NA
2014:Q3	0.0110	0.0210	NA	0.0110	0.0215	NA	0.0110	0.0208	NA
2014:Q4	0.0110	0.0212	NA	0.0110	0.0209	NA	0.0110	0.0213	NA
2015:Q1	0.0110	0.0210	NA	0.0110	0.0196	NA	0.0110	0.0232	NA
2015:Q2	0.0110	0.0211	NA	0.0110	0.0214	NA	0.0110	0.0213	NA
2015:Q3	0.0110	0.0216	NA	0.0110	0.0207	NA	0.0110	0.0209	NA
2015:Q4	0.0110	0.0215	NA	0.0110	0.0208	NA	0.0110	0.0214	NA
2016:Q1	0.0110	0.0211	NA	0.0110	0.0211	NA	0.0110	0.0206	NA
2016:Q2	0.0110	0.0211	0.1931	0.0110	0.0210	0.2506	0.0110	0.0213	0.0728
Consumption	0.0105	0.0107	0.0419	0.0112	0.0255	0.0140	0.0110	0.0210	0.2421
Investment	0.0100	0.0220	0.0052	0.0098	0.0225	0.0078	0.0088	0.0077	0.0713
Exports	0.0111	0.0222	0.3556	0.0111	0.0216	0.2368	0.0110	0.0212	0.0131
Imports	0.0111	0.0220	0.2380	0.0111	0.0217	0.0829	0.0110	0.0212	0.1196
Labor Productivy Growth	0.0109	0.0214	0.1468	0.0109	0.0184	0.4080	0.0110	0.0211	0.3156
Employment Share	0.0105	0.0159	0.0195	0.0104	0.0178	0.0000	0.0110	0.0212	0.1655

Table D.17: Estimated $\boldsymbol{v}$ weights considering 2016:Q2 as treatment period, last covariates and a ridge penalty.

	SC(B) all	**SC(B) half**	**SC(B) one**	**SC all**	**SC half**	**SC one**	**DSC all**	**DSC half**	**DSC one**
2014:Q1	0.0110	0.0213	NA	0.0110	0.0214	NA	0.0110	0.0211	NA
2014:Q2	0.0110	0.0213	NA	0.0110	0.0219	NA	0.0110	0.0216	NA
2014:Q3	0.0110	0.0197	NA	0.0110	0.0218	NA	0.0110	0.0214	NA
2014:Q4	0.0110	0.0211	NA	0.0110	0.0215	NA	0.0110	0.0196	NA
2015:Q1	0.0110	0.0211	NA	0.0110	0.0219	NA	0.0110	0.0216	NA
2015:Q2	0.0110	0.0193	NA	0.0110	0.0214	NA	0.0110	0.0216	NA
2015:Q3	0.0110	0.0208	NA	0.0110	0.0213	NA	0.0110	0.0218	NA
2015:Q4	0.0110	0.0208	NA	0.0110	0.0214	NA	0.0110	0.0217	NA
2016:Q1	0.0110	0.0208	NA	0.0110	0.0218	NA	0.0110	0.0216	NA
2016:Q2	0.0110	0.0208	0.0821	0.0110	0.0215	0.4734	0.0110	0.0218	0.3156
Consumption	0.0110	0.0209	0.0218	0.0110	0.0209	0.1972	0.0110	0.0198	0.0692
Investment	0.0110	0.0181	0.0035	0.0088	0.0110	0.0000	0.0088	0.0098	0.0009
Exports	0.0110	0.0222	0.2539	0.0110	0.0215	0.0425	0.0110	0.0205	0.1122
Imports	0.0110	0.0209	0.2608	0.0110	0.0212	0.1566	0.0110	0.0210	0.2679
Labor Productivity Growth	0.0121	0.0256	0.3718	0.0110	0.0212	0.0121	0.0110	0.0199	0.0085
Employment Share	0.0110	0.0180	0.0061	0.0110	0.0222	0.1182	0.0110	0.0188	0.2257

Appendix D.3. Estimated ω Weights

As expected, including regularization term leads to a bigger dispersion in the estimated unit weights for some methods. In particular, in Tables D.18-D.21, we see that for all cases considered, there is an increase in dispersion for the SC and DSC (and, consequently, SDID) methods without matching on covariates as opposed to when no penalty term is taken into account. Moreover, for some specifications, the increase in dispersion is also observed for the SC and the DSC methods with matching on covariates and considering only one pre-treatment period.

Table D.18: Estimated $\boldsymbol{\omega}$ weights considering 2016:Q3 as treatment period, mean of covariates and a penalty term.

	SC(B) all	**SC(B) half**	**SC(B) one**	**SC no cov**	**SC all**	**SC half**	**SC one**	**DSC no cov**	**DSC all**	**DSC half**	**DSC one**
Australia	0.0009	0.0000	0.0000	0.0009	0.0004	0.0000	0.0000	0.0017	0.0000	0.0000	0.0000
Austria	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Belgium	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Canada	0.0000	0.0000	0.0001	0.1214	0.0000	0.0000	0.0000	0.1241	0.0006	0.0004	0.0000
Finland	0.0000	0.0000	0.0000	0.0284	0.0000	0.0000	0.0000	0.0239	0.0000	0.0000	0.0000
France	0.0000	0.0000	0.0000	0.0558	0.0000	0.0000	0.0000	0.0614	0.0000	0.0000	0.0000
Germany	0.1247	0.1399	0.0221	0.0273	0.1236	0.1001	0.0228	0.0378	0.0131	0.0659	0.0366
Hungary	0.0553	0.0000	0.0000	0.1702	0.0557	0.0000	0.0000	0.1794	0.0603	0.0003	0.0000
Iceland	0.0765	0.1321	0.0002	0.0000	0.0754	0.0910	0.0188	0.0043	0.0067	0.0720	0.0610
Ireland	0.0137	0.0000	0.0000	0.0701	0.0136	0.0000	0.0000	0.0649	0.0370	0.0413	0.0000
Italy	0.1531	0.1714	0.1529	0.0588	0.1545	0.1730	0.2221	0.0593	0.2392	0.2012	0.1764
Japan	0.0000	0.0000	0.0000	0.1519	0.0000	0.0000	0.0000	0.1469	0.0000	0.0004	0.0000
Korea	0.0000	0.0000	0.0000	0.0037	0.0000	0.0000	0.0000	0.0030	0.0000	0.0000	0.0000
Luxembourg	0.0000	0.0305	0.0251	0.0000	0.0000	0.0260	0.0536	0.0000	0.0000	0.0000	0.0337
Netherlands	0.0000	0.0000	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
New Zealand	0.0669	0.0000	0.0644	0.0567	0.0649	0.0000	0.1184	0.0641	0.1567	0.0000	0.0000
Norway	0.0014	0.0000	0.0000	0.1054	0.0005	0.0000	0.0000	0.0964	0.0000	0.0000	0.0000
Portugal	0.0000	0.0000	0.0085	0.0079	0.0000	0.0001	0.0000	0.0026	0.0000	0.0000	0.0000
Slovak Republic	0.0000	0.0000	0.0000	0.0038	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Spain	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Sweden	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Switzerland	0.0009	0.0000	0.1240	0.0035	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
United States	0.5065	0.5261	0.6022	0.1340	0.5112	0.6097	0.5643	0.1303	0.4865	0.6184	0.6923

Table D.19: Estimated $\boldsymbol{\omega}$ weights considering 2016:Q3 as treatment period, last covariates and a penalty term.

	SC(B) all	**SC(B) half**	**SC(B) one**	**SC no cov**	**SC all**	**SC half**	**SC one**	**DSC no cov**	**DSC all**	**DSC half**	**DSC one**
Australia	0.0891	0.0000	0.0000	0.0009	0.0898	0.0000	0.0007	0.0017	0.0000	0.0000	0.0007
Austria	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
Belgium	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
Canada	0.1522	0.1158	0.0196	0.1214	0.1421	0.0843	0.1248	0.1241	0.0951	0.0555	0.1285
Finland	0.0000	0.0000	0.0000	0.0284	0.0000	0.0001	0.0000	0.0239	0.0000	0.0000	0.0000
France	0.0000	0.0000	0.0000	0.0558	0.0000	0.0001	0.0000	0.0614	0.0000	0.0000	0.0000
Germany	0.0005	0.0645	0.0001	0.0273	0.0020	0.0726	0.0327	0.0378	0.0001	0.0000	0.0365
Hungary	0.0815	0.0001	0.0000	0.1702	0.0895	0.0004	0.0000	0.1794	0.1496	0.1029	0.0000
Iceland	0.0020	0.0146	0.0000	0.0000	0.0064	0.0226	0.0966	0.0043	0.0012	0.0425	0.0782
Ireland	0.0000	0.0000	0.0000	0.0701	0.0000	0.0001	0.0000	0.0649	0.0000	0.0000	0.0000
Italy	0.0789	0.0082	0.0344	0.0588	0.0730	0.0325	0.0740	0.0593	0.1297	0.1510	0.0995
Japan	0.0648	0.0000	0.0069	0.1519	0.0723	0.0052	0.0816	0.1469	0.0598	0.0908	0.0513
Korea	0.0000	0.0000	0.0000	0.0037	0.0000	0.0000	0.0000	0.0030	0.0000	0.0000	0.0000
Luxembourg	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
Netherlands	0.0000	0.0000	0.0000	0.0000	0.0001	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
New Zealand	0.1041	0.1978	0.3050	0.0567	0.0891	0.2199	0.2104	0.0641	0.1638	0.1191	0.2162
Norway	0.0000	0.0000	0.0000	0.1054	0.0001	0.0001	0.0000	0.0964	0.0000	0.0000	0.0000
Portugal	0.1426	0.2855	0.2788	0.0079	0.1377	0.2361	0.1472	0.0026	0.0532	0.0000	0.1331
Slovak Republic	0.0000	0.0000	0.0000	0.0038	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
Spain	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Sweden	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
Switzerland	0.0000	0.0000	0.0000	0.0035	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
United States	0.2842	0.3133	0.3552	0.1340	0.2973	0.3253	0.2320	0.1303	0.3476	0.4382	0.2559

Table D.20: Estimated $\boldsymbol{\omega}$ weights considering 2016:Q2 as treatment period, mean of covariates and a penalty term.

	SC(B) all	**SC(B) half**	**SC(B) one**	**SC no cov**	**SC all**	**SC half**	**SC one**	**DSC no cov**	**DSC all**	**DSC half**	**DSC one**
Australia	0.0020	0.0000	0.0000	0.0040	0.0020	0.0000	0.0000	0.0017	0.0000	0.0000	
Austria	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	
Belgium	0.0001	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0001	
Canada	0.0003	0.0000	0.0000	0.1163	0.0002	0.0000	0.0001	0.1208	0.0043	0.0024	
Finland	0.0000	0.0000	0.0000	0.0253	0.0000	0.0000	0.0000	0.0209	0.0001	0.0000	
France	0.0000	0.0000	0.0000	0.0533	0.0000	0.0000	0.0000	0.0598	0.0000	0.0000	
Germany	0.1255	0.1378	0.0013	0.0305	0.1219	0.0994	0.0534	0.0410	0.0087	0.0376	
Hungary	0.0572	0.0000	0.0000	0.1703	0.0577	0.0000	0.0000	0.1785	0.0575	0.0188	
Iceland	0.0742	0.1268	0.0000	0.0019	0.0714	0.0877	0.0390	0.0058	0.0057	0.0733	
Ireland	0.0132	0.0000	0.0000	0.0719	0.0124	0.0000	0.0000	0.0669	0.0377	0.0365	
Italy	0.1489	0.1708	0.1427	0.0601	0.1489	0.1717	0.2031	0.0598	0.2423	0.1700	
Japan	0.0000	0.0000	0.0000	0.1536	0.0000	0.0000	0.0000	0.1479	0.0007	0.0547	
Korea	0.0000	0.0000	0.0000	0.0044	0.0000	0.0000	0.0000	0.0029	0.0000	0.0000	
Luxembourg	0.0001	0.0323	0.0299	0.0000	0.0001	0.0268	0.0475	0.0000	0.0003	0.0006	
Netherlands	0.0001	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0002	0.0001	
New Zealand	0.0649	0.0000	0.0859	0.0574	0.0656	0.0000	0.1206	0.0663	0.1605	0.0002	
Norway	0.0029	0.0000	0.0000	0.1035	0.0021	0.0000	0.0000	0.0947	0.0001	0.0001	
Portugal	0.0000	0.0000	0.0507	0.0090	0.0000	0.0001	0.0001	0.0037	0.0001	0.0000	
Slovak Republic	0.0000	0.0000	0.0000	0.0028	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
Spain	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
Sweden	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	
Switzerland	0.0025	0.0000	0.1004	0.0035	0.0029	0.0000	0.0000	0.0000	0.0001	0.0001	
United States	0.5080	0.5323	0.5891	0.1324	0.5145	0.6141	0.5361	0.1295	0.4814	0.6053	

Table D.21: Estimated ω weights considering 2016:Q2 as treatment period, last covariates and a penalty term.

	SC(B) all	**SC(B) half**	**SC(B) one**	**SC no cov**	**SC all**	**SC half**	**SC one**	**DSC no cov**	**DSC all**	**DSC half**	**DSC one**
Australia	0.1283	0.0001	0.1568	0.0040	0.1294	0.0001	0.0000	0.0017	0.0000	0.0000	0.0005
Austria	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002
Belgium	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002
Canada	0.1890	0.2293	0.1418	0.1163	0.1909	0.2437	0.0000	0.1208	0.1632	0.1454	0.0017
Finland	0.0000	0.0000	0.0003	0.0253	0.0000	0.0000	0.0000	0.0209	0.0000	0.0000	0.0003
France	0.0000	0.0000	0.0001	0.0533	0.0000	0.0000	0.0000	0.0598	0.0000	0.0000	0.0002
Germany	0.0045	0.0507	0.0003	0.0305	0.0020	0.0396	0.0000	0.0410	0.0000	0.0000	0.0004
Hungary	0.0631	0.0000	0.0000	0.1703	0.0683	0.0000	0.0000	0.1785	0.1027	0.0000	0.0001
Iceland	0.0576	0.0764	0.0313	0.0019	0.0488	0.0781	0.0000	0.0058	0.0153	0.0873	0.0376
Ireland	0.0000	0.0000	0.0000	0.0719	0.0000	0.0000	0.0000	0.0669	0.0217	0.0460	0.0001
Italy	0.0889	0.0000	0.0003	0.0601	0.0755	0.0000	0.0000	0.0598	0.0966	0.0889	0.0002
Japan	0.0822	0.0002	0.0206	0.1536	0.0944	0.0381	0.0779	0.1479	0.0770	0.1397	0.0819
Korea	0.0000	0.0000	0.0000	0.0044	0.0001	0.0000	0.0000	0.0029	0.0000	0.0000	0.0002
Luxembourg	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001
Netherlands	0.0000	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002
New Zealand	0.0053	0.0001	0.0055	0.0574	0.0086	0.0004	0.2980	0.0663	0.1196	0.0000	0.2899
Norway	0.0000	0.0001	0.0001	0.1035	0.0001	0.0000	0.0000	0.0947	0.0000	0.0000	0.0002
Portugal	0.1666	0.3314	0.3282	0.0090	0.1590	0.2795	0.2219	0.0037	0.1044	0.0825	0.2215
Slovak Republic	0.0000	0.0000	0.0000	0.0028	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002
Spain	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001
Sweden	0.0000	0.0000	0.0001	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002
Switzerland	0.0000	0.0000	0.0001	0.0035	0.0002	0.0000	0.0218	0.0000	0.0000	0.0000	0.0179
United States	0.2144	0.3113	0.3142	0.1324	0.2223	0.3204	0.3802	0.1295	0.2996	0.4101	0.3462

Appendix D.4. In-sample Placebo Analysis

In Table D.22 we summarize the in-sample placebo analysis with penalized weights $\boldsymbol{\omega}$.

- Overall, the penalty term plays little role in this exercise. The differences between the results in the main text and the ones with penalty are usually of the order 10^{-4}.
- One notable exception is SDID(i) with covariates, where penalization has some positive effect on the RMSE measure.
- For the last specification reported in that table, the numerical optimization failed for a few time periods. For this reason, we leave empty the corresponding cells for methods with covariates.

Table D.22: Placebo analysis for 2010:Q1-2014:Q4 with penalty.

	SC(B)	**SC**	**SC cov.**	**DSC**	**DSC cov.**	**SDID (i)**	**SDID cov.(i)**	**SDID (ii)**	**SDID cov.(ii)**	**SDID (iii)**	**SDID cov.(iii)**
Mean of covariates - All pre-treatments											
RMSE	0.0108	0.0092	0.0105	0.0090	0.0117	0.0066	0.0047	0.0131	0.0095	0.0133	0.0096
MAB	0.0090	0.0075	0.0090	0.0074	0.0107	0.0036	0.0036	0.0106	0.0082	0.0108	0.0083
MedAB	0.0104	0.0074	0.0102	0.0072	0.0108	0.0015	0.0022	0.0070	0.0073	0.0079	0.0074
Last covariates - All pre-treatments											
RMSE	0.0141	0.0092	0.0130	0.0090	0.0137	0.0066	0.0053	0.0131	0.0118	0.0133	0.0118
MAB	0.0123	0.0075	0.0114	0.0074	0.0121	0.0036	0.0035	0.0106	0.0100	0.0108	0.0101
MedAB	0.0140	0.0074	0.0126	0.0072	0.0141	0.0015	0.0021	0.0070	0.0081	0.0079	0.0083
Mean of covariates - Half of the pre-treatments											
RMSE	0.0123	0.0092	0.0118	0.0090	0.0114	0.0066	0.0047	0.0131	0.0088	0.0133	0.0088
MAB	0.0112	0.0075	0.0108	0.0074	0.0102	0.0036	0.0035	0.0106	0.0072	0.0108	0.0073
MedAB	0.0107	0.0074	0.0105	0.0072	0.0103	0.0015	0.0030	0.0070	0.0057	0.0079	0.0057
Last covariates - Half of the pre-treatments											
RMSE	0.0214	0.0092	0.0165	0.0090	0.0094	0.0066	0.0054	0.0131	0.0109	0.0133	0.0109
MAB	0.0187	0.0075	0.0136	0.0074	0.0078	0.0036	0.0033	0.0106	0.0094	0.0108	0.0095
MedAB	0.0198	0.0074	0.0141	0.0072	0.0076	0.0015	0.0018	0.0070	0.0082	0.0079	0.0082
Mean of covariates - One of the pre-treatments											
RMSE	0.0084	0.0092	0.0086	0.0090	0.0098	0.0066	0.0042	0.0131	0.0074	0.0133	0.0075
MAB	0.0072	0.0075	0.0076	0.0074	0.0091	0.0036	0.0034	0.0106	0.0061	0.0108	0.0061
MedAB	0.0072	0.0074	0.0080	0.0072	0.0092	0.0015	0.0028	0.0070	0.0051	0.0079	0.0051
Last covariates - One of the pre-treatments											
RMSE		0.0092		0.0090		0.0066		0.0131		0.0133	
MAB		0.0075		0.0074		0.0036		0.0106		0.0108	
MedAB		0.0074		0.0072		0.0015		0.0070		0.0079	

Appendix E. Robustness Exercise: Excluding the US from the Donor Pool

In the same year as the Brexit vote occurred, another (possibly) important economic event happened: the American elections, where the (now) ex-president Donald Trump was elected. There are several ways in which this event might have affected the economy of the US. For instance, there was a shift in the trade policies. The possible effect of Trump on the American economy may render the inclusion of the US in the donor pool of the SC methods used in this study questionable. Therefore, we consider a robustness exercise, where we repeat all the estimations from the main text for the case where the US is not a part of the donor pool. We note that a similar sensitivity analysis was not done by Born et al. (2019).

Appendix E.1. Estimated Treatment Effects

Tables E.23-E.25 show the estimated treatment effects when considering all pre-treatment periods, half of the pre-treatment periods, and one pre-treatment period, respectively.

- All of these specifications share one common feature: compared to the estimates when considering the US in the donor pool, the effects obtained with methods without matching on covariates are very similar.
- When considering covariates, there does not seem to be a clear pattern when comparing to the estimates including the US in the donor pool.
- When considering all pre-treatment periods (Table E.23), the estimates for the methods without covariates are always bigger than the effects obtained with the SC(B) method, independently of the specification, and also, importantly, are bigger than the original effect estimated by Born et al. (2019). When taking into account covariates (last covariates), only the estimated effects under the DSC and SDID methods are smaller compared to the SC(B).
- The conclusions change substantially when half of the pre-treatment periods are considered. In this case, when compared to the estimations obtained with the SC(B) method, the other methods deliver a smaller estimate when the mean of the covariates is considered, while they deliver a bigger estimate when the last covariates are taken into account (with the exception for the SC method using the treatment period 2016:Q3). However, it is important to note that all remaining methods still point out to a bigger estimated effect when compared to the initial estimation of Born et al. (2019).
- Finally, when considering only one pre-treatment period (Table E.25), all methods point out to a bigger estimate than that obtained via SC(B), independently of the specification. Moreover, all remaining methods without covariates point out to a bigger estimated effect than that of Born et al. (2019)

Table E.23: The estimated percentage differences in GDP between the counterfactuals and the UK including all pre-treatment periods as predictors.

	SC(B)	**SC**	**SC cov.**	**DSC**	**DSC cov.**	**SDID (i)**	**SDID cov. (i)**	**SDID (ii)**	**SDID cov. (ii)**	**SDID (iii)**	**SDID cov. (iii)**
Mean of covariates - Treatment period 2016:Q2											
2018:Q4	2.65	3.13	3.09	3.07	3.44	3.20	3.16	3.00	2.83	3.00	2.83
2019:Q4	3.76	4.19	4.13	4.14	4.38	4.27	4.10	4.07	3.76	4.07	3.76
Mean of covariates - Treatment period 2016:Q3											
2018:Q4	2.55	3.07	3.04	3.03	2.13	2.82	2.00	2.86	2.04	2.86	2.04
2019:Q4	3.67	4.12	4.07	4.10	3.07	3.88	2.94	3.93	2.98	3.93	2.98
Last covariates - Treatment period 2016:Q2											
2018:Q4	1.91	3.13	3.11	3.07	3.21	3.20	3.24	3.00	3.01	3.00	3.01
2019:Q4	2.91	4.19	4.16	4.14	4.24	4.27	4.27	4.07	4.04	4.07	4.04
Last covariates - Treatment period 2016:Q3											
2018:Q4	1.84	3.07	3.14	3.03	1.51	2.82	2.48	2.86	2.58	2.86	2.58
2019:Q4	2.77	4.12	4.20	4.10	2.55	3.88	3.51	3.93	3.61	3.93	3.61

Table E.24: The estimated percentage differences in GDP between the counterfactuals and the UK including half of the pre-treatment periods as predictors.

	SC(B)	**SC**	**SC cov.**	**DSC**	**DSC cov.**	**SDID (i)**	**SDID cov. (i)**	**SDID (ii)**	**SDID cov. (ii)**	**SDID (iii)**	**SDID cov. (iii)**
Mean of covariates - Treatment period 2016:Q2											
2018:Q4	3.41	3.13	3.06	3.07	3.28	3.20	3.19	3.00	2.98	3.00	2.98
2019:Q4	4.49	4.19	4.08	4.14	4.29	4.27	4.20	4.07	4.00	4.07	4.00
Mean of covariates - Treatment period 2016:Q3											
2018:Q4	3.10	3.07	3.03	3.03	3.13	2.82	2.75	2.86	2.78	2.86	2.78
2019:Q4	4.19	4.12	4.22	4.10	4.15	3.88	3.77	3.93	3.80	3.93	3.80
Last covariates - Treatment period 2016:Q2											
2018:Q4	1.79	3.13	3.18	3.07	3.50	3.20	3.42	3.00	3.22	3.00	3.22
2019:Q4	2.69	4.19	4.09	4.14	4.55	4.27	4.46	4.07	4.26	4.07	4.26
Last covariates - Treatment period 2016:Q3											
2018:Q4	1.96	3.07	1.79	3.03	3.19	2.82	2.79	2.86	2.81	2.86	2.81
2019:Q4	2.91	4.12	2.73	4.10	4.22	3.88	3.81	3.93	3.84	3.93	3.84

Table E.25: The estimated percentage differences in GDP between the counterfactuals and the UK including one of the pre-treatment periods as predictors.

	SC(B)	**SC**	**SC cov.**	**DSC**	**DSC cov.**	**SDID (i)**	**SDID cov. (i)**	**SDID (ii)**	**SDID cov. (ii)**	**SDID (iii)**	**SDID cov. (iii)**
Mean of covariates - Treatment period 2016:Q2											
2018:Q4	1.91	3.13	2.48	3.07	2.75	3.20	3.18	3.00	2.78	3.00	2.78
2019:Q4	3.19	4.19	3.68	4.14	3.84	4.27	4.27	4.07	3.86	4.07	3.86
Mean of covariates - Treatment period 2016:Q3											
2018:Q4	1.56	3.07	1.85	3.03	1.73	2.82	1.67	2.86	1.75	2.86	1.75
2019:Q4	2.83	4.12	3.10	4.10	2.88	3.88	2.81	3.93	2.89	3.93	2.89
Last covariates - Treatment period 2016:Q2											
2018:Q4	1.84	3.13	2.66	3.07	2.72	3.20	3.28	3.00	2.81	3.00	2.81
2019:Q4	2.96	4.19	3.77	4.14	3.77	4.27	4.32	4.07	3.85	4.07	3.85
Last covariates - Treatment period 2016:Q3											
2018:Q4	1.49	3.07	1.43	3.03	1.57	2.82	1.61	2.86	1.69	2.86	1.69
2019:Q4	2.66	4.12	2.39	4.10	2.72	3.88	2.76	3.93	2.84	3.93	2.84

Appendix E.2. Estimated ω Weights

For the sake of brevity, we present the estimated unit weights according to the different methods only for the treatment period 2016:Q3, taking into account no covariates, the mean of the covariates, and the last covariates. The conclusions presented here are the same for the case when the treatment period is set to 2016:Q2.

- For the estimates without covariates, the non-zero weights are assigned to the same remaining countries in the donor pool as for the previous results when the US was not excluded from the control units, and the only change is in the magnitude of the weights.

- For the estimates with covariates, when compared to the complete donor pool benchmark case, positive weights are assigned to other countries, including Australia, Canada, Iceland, New Zealand, and Norway. Therefore, the estimates without covariates seem to be less sensitive to the inclusion or not of the US in the donor pool.

Table E.26: Estimated $\boldsymbol{\omega}$ weights considering 2016:Q3 as treatment period and mean of covariates.

	SC(B) all	**SC(B) half**	**SC(B) one**	**SC no cov**	**SC all**	**SC half**	**SC one**	**DSC no cov**	**DSC all**	**DSC half**	**DSC one**
Australia	0.1224	0.0012	0.2687	0.0067	0.0002	0.0000	0.1538	0.0012	0.0000	0.0000	0.1151
Austria	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Belgium	0.0001	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
Canada	0.1023	0.0000	0.0000	0.2569	0.2835	0.0005	0.0001	0.2836	0.2176	0.3656	0.0000
Finland	0.0001	0.0000	0.0000	0.0242	0.0357	0.0000	0.0000	0.0141	0.0000	0.0000	0.0000
France	0.0001	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
Germany	0.0008	0.0000	0.0000	0.0021	0.0000	0.0376	0.0001	0.0000	0.0017	0.0000	0.0000
Hungary	0.0004	0.0000	0.0000	0.2118	0.2121	0.0003	0.0000	0.2344	0.0001	0.1615	0.0000
Iceland	0.0788	0.1602	0.0000	0.0000	0.0005	0.1130	0.1237	0.0004	0.0489	0.0365	0.1205
Ireland	0.0414	0.0000	0.0000	0.0724	0.0684	0.0005	0.0000	0.0625	0.0678	0.0712	0.0000
Italy	0.1520	0.1433	0.1103	0.0192	0.0000	0.1678	0.2327	0.0051	0.3535	0.1020	0.2633
Japan	0.1255	0.1631	0.0000	0.2104	0.2169	0.0000	0.0297	0.2159	0.0160	0.2165	0.0537
Korea	0.0000	0.0000	0.0000	0.0029	0.0000	0.0000	0.0000	0.0022	0.0000	0.0000	0.0000
Luxembourg	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
Netherlands	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
New Zealand	0.2297	0.4606	0.2698	0.0000	0.0000	0.4858	0.3079	0.0000	0.2938	0.0465	0.3463
Norway	0.0000	0.0000	0.0000	0.1699	0.1587	0.0000	0.0000	0.1587	0.0000	0.0000	0.0000
Portugal	0.1464	0.0716	0.3512	0.0201	0.0237	0.1937	0.1519	0.0218	0.0003	0.0000	0.1010
Slovak Republic	0.0000	0.0000	0.0000	0.0034	0.0000	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
Spain	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Sweden	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Switzerland	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table E.27: Estimated $\boldsymbol{\omega}$ weights considering 2016:Q3 as treatment period and last covariates.

	SC(B) all	**SC(B) half**	**SC(B) one**	**SC no cov**	**SC all**	**SC half**	**SC one**	**DSC no cov**	**DSC all**	**DSC half**	**DSC one**
Australia	0.0177	0.0000	0.2759	0.0067	0.0004	0.0000	0.0000	0.0012	0.0000	0.0000	0.1293
Austria	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
Belgium	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
Canada	0.3927	0.3212	0.0000	0.2569	0.2834	0.4181	0.4434	0.2836	0.3366	0.3651	0.0040
Finland	0.0000	0.0000	0.0000	0.0242	0.0173	0.0002	0.0000	0.0141	0.0000	0.0000	0.0001
France	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0001
Germany	0.0000	0.0000	0.0000	0.0021	0.0000	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000
Hungary	0.0000	0.0002	0.0000	0.2118	0.2133	0.0010	0.0000	0.2344	0.1641	0.1695	0.0000
Iceland	0.0754	0.1198	0.0000	0.0000	0.0022	0.1067	0.1117	0.0004	0.0270	0.0320	0.1194
Ireland	0.0000	0.0015	0.0000	0.0724	0.0694	0.0006	0.0000	0.0625	0.0000	0.0733	0.0000
Italy	0.0963	0.1782	0.1000	0.0192	0.0000	0.1092	0.2280	0.0051	0.1172	0.0945	0.2750
Japan	0.0814	0.0925	0.0873	0.2104	0.2139	0.0824	0.0000	0.2159	0.0904	0.2238	0.0521
Korea	0.0000	0.0000	0.0000	0.0029	0.0000	0.0001	0.0000	0.0022	0.0000	0.0000	0.0000
Luxembourg	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
Netherlands	0.0000	0.0000	0.0000	0.0000	0.0006	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
New Zealand	0.2010	0.2520	0.2779	0.0000	0.0027	0.1631	0.1652	0.0000	0.1751	0.0417	0.3276
Norway	0.0000	0.0000	0.0000	0.1699	0.1646	0.0001	0.0000	0.1587	0.0000	0.0000	0.0000
Portugal	0.1355	0.0344	0.2589	0.0201	0.0318	0.1170	0.0517	0.0218	0.0895	0.0000	0.0921
Slovak Republic	0.0000	0.0000	0.0000	0.0034	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
Spain	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Sweden	0.0000	0.0000	0.0000	0.0000	0.0001	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
Switzerland	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000

Appendix E.3. In-sample Placebo Analysis

Table E.28 shows the results obtained when performing the in-sample placebo analysis across time periods after removing the US as one of the control units.

- The results are largely in line with the previous findings in the benchmark case.
- In general, the SC(B) performs the worst, with the exception of the specification where one pre-treatment period and the last covariates are considered, in which case the SC with covariates performs the worst.
- Across all specifications, the SDID or the SDID with covariates performs the best, with the differences in the performance between these two methods being most of the time almost negligible.

Table E.28: Placebo analysis for 2010:Q1-2014:Q4.

	SC(B)	**SC**	**SC cov.**	**DSC**	**DSC cov.**	**SDID (i)**	**SDID cov.(i)**	**SDID (ii)**	**SDID cov.(ii)**	**SDID (iii)**	**SDID cov.(iii)**
Mean of covariates - All pre-treatments											
RMSE	0.0170	0.0093	0.0093	0.0091	0.0096	0.0072	0.0066	0.0142	0.0140	0.0142	0.0140
MAB	0.0146	0.0073	0.0070	0.0071	0.0072	0.0043	0.0043	0.0119	0.0121	0.0119	0.0121
MedAB	0.0197	0.0056	0.0041	0.0052	0.0052	0.0027	0.0026	0.0112	0.0107	0.0115	0.0107
Last covariates - All pre-treatments											
RMSE	0.0163	0.0093	0.0073	0.0091	0.0091	0.0072	0.0071	0.0142	0.0138	0.0142	0.0139
MAB	0.0143	0.0073	0.0059	0.0071	0.0070	0.0043	0.0042	0.0119	0.0112	0.0119	0.0113
MedAB	0.0138	0.0056	0.0046	0.0052	0.0044	0.0027	0.0020	0.0112	0.0116	0.0115	0.0116
Mean of covariates - Half of the pre-treatments											
RMSE	0.0158	0.0093	0.0140	0.0091	0.0090	0.0072	0.0075	0.0142	0.0148	0.0142	0.0148
MAB	0.0133	0.0073	0.0105	0.0071	0.0070	0.0043	0.0047	0.0119	0.0125	0.0119	0.0125
MedAB	0.0140	0.0056	0.0078	0.0052	0.0059	0.0027	0.0029	0.0112	0.0114	0.0115	0.0113
Last covariates - Half of the pre-treatments											
RMSE	0.0165	0.0093	0.0096	0.0091	0.0087	0.0072	0.0068	0.0142	0.0142	0.0142	0.0142
MAB	0.0143	0.0073	0.0083	0.0071	0.0071	0.0043	0.0042	0.0119	0.0113	0.0119	0.0114
MedAB	0.0144	0.0056	0.0075	0.0052	0.0062	0.0027	0.0021	0.0112	0.0091	0.0115	0.0094
Mean of covariates - One of the pre-treatments											
RMSE	0.0073	0.0093	0.0182	0.0091	0.0151	0.0072	0.0055	0.0142	0.0133	0.0142	0.0134
MAB	0.0061	0.0073	0.0159	0.0071	0.0124	0.0043	0.0046	0.0119	0.0115	0.0119	0.0117
MedAB	0.0059	0.0056	0.0190	0.0052	0.0127	0.0027	0.0043	0.0112	0.0094	0.0115	0.0097
Last covariates - One of the pre-treatments											
RMSE	0.0154	0.0093	0.0148	0.0091	0.0131	0.0072	0.0053	0.0142	0.0130	0.0142	0.0130
MAB	0.0129	0.0073	0.0124	0.0071	0.0101	0.0043	0.0042	0.0119	0.0111	0.0119	0.0112
MedAB	0.0134	0.0056	0.0129	0.0052	0.0081	0.0027	0.0026	0.0112	0.0101	0.0115	0.0103

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